CBSE Class 10th Maths

Value Based Questions

CHAPTER –10

CIRCLE

1. **There are 3 villages A, B and C such that the distance from A to B is 7 km, from B to C is 5 km and from C to A is 8 km. The gram pradhan wants to dig a well in such a way that the distance from each villages are equal. What should be the location of well? Which value is depicted by gram pradhan?**

   Ans. A, B, C will lie on the circumference of the circle and location of well will be at the centre of the circle. Social, Honesty, Equality.

2. **People of village wants to construct a road nearest to a circular village Rampur. The road cannot pass through the village. But the people wants that road should be at the shortest distance from the center of the village**

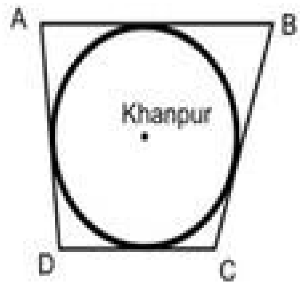
   (i) which road will be the nearest to the center of village?

   (ii) which value is depicted by the people of village?

   Ans.

   i. Tangent of the circle
   ii. Economical

3. **Four roads have to be constructed by touching village Khanpur in circular shape of radius 1700 m in the following manner.**

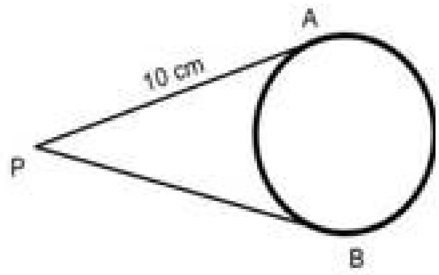


   Savita got contract to construct the roads AB and CD while Vijay got contract to construct AD and BC. Prove that AB + CD = AD + BC. Which value is depicted by the contractor?

   Ans. Gender equality

4. **Two roads starting from P are touching a circular path at A and B. Sarita ran from P**

to A 10 km and Ramesh ran from P to B.



(i) If Sarita wins the race than how much distance Ramesh ran?

(ii) Which value is depicted?

Ans. 10 km, Gender equality

5. **A farmer wants to divide a sugarcane of 7 ft length between his son and daughter equally. Divide it Geometrically, considering sugarcane as a line of 7 cm, using construction.**

(i) Find the length of each part.

(ii) Which value is depicted?

Ans. 3.5 ft, Gender equality.

CBSE Class 10 Mathematics

Important Questions

Chapter 10

Circles

1 Marks Questions

1. How many tangents can a circle have?

Ans. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

2. The perimeter of a sector of a circle of radius 8 cm is 25m, what is area of sector?

(a) $50cm^2$

(b) $42cm^2$

(c) $52cm^2$

(d) none of these

Ans. (a) $50cm^2$

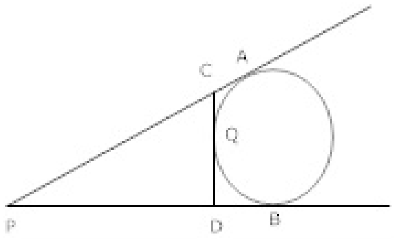
3. In figure given below PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PA = 10cm and DQ = 2cm. What is length of PC?

(a) 8 cm

(b) 7 cm

(c) 4 cm

(d) none of these



Ans. (a) 8 cm

4. Tangent of circle intersect the circle

(a) Only one point

(b) Two points

(c) Three points

(d) None of these

Ans. (a) Only one point

5. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25cm. The radius of the circle is

(a) 7 cm

(b) 12 cm

(c) 15 cm

(d) 24.5 cm

Ans. (a) 7 cm

6. How many tangents can a circle have?

(a) 1

(b) 2

(c) 0

(d) infinite

Ans. d) infinite

7. If PA and PB are tangents from a point P lying outside the circle such that PA = 10 cm and $\angle APB = 60^\circ$. Find length of chord AB.

(a) 10 cm

(b) 20 cm

(c) 30 cm

(d) 40 cm

Ans. (a) 10cm

8. A tangent PQ at a point P to a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 13cm, then length of PQ is

(a) 11 cm

(b) 12 cm

(c) 10 cm

(d) None of these

Ans. (b) 12 cm

9. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle POA$ is equal to

(a) 50°

(b) 60°

(c) 70°

(d) 80°

Ans. (a) 50°

10. How many tangents can a circle have?

(a) 1

(b) 2

(c) 0

(d) infinite

Ans. (d) infinite

11. If PA and PB are tangents from a point P lying outside the circle such that PA = 10 cm and $\angle APB = 60°$. Find length of chord AB.

(a) 10 cm

(b) 20 cm

(c) 30 cm

(d) 40 cm

Ans. (a) 10 cm

12. A tangent PQ at a point P to a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 13cm, then length of PQ is

(a) 11 cm

(b) 12 cm

(c) 10 cm

(d) None of these

Ans. (b) 12 cm

13. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of $80°$, then $\angle POA$ is equal to

(a) 50°

(b) 60°

(c) 70°

(d) 80°

Ans. (a) 50°

14. The length of tangent drawn to a circle with radius 3 cm from a point 5 cm from the centre of the circle is

(a) 6 cm

(b) 8 cm

(c) 4 cm

(d) 7 cm

Ans. (c) 4 cm

15. A circle touches all the four sides of a quadrilateral ABCD whose sides AB = 6 cm, BC = 7 cm, CD = 4 cm,then AD = ____

(a) 2 cm

(b) 3 cm

(c) 5 cm

(d) 6 cm

Ans. (b) 3 cm

16. If a point lies on a circle, then what will be the number of tangents drawn from that point to the circle?

(a) 1

(b) 2

(c) 3

(d) infinite

Ans. (a) 1

17. A quadrilateral ABCD is drawn to circumscribe a circle IF AB = 4 cm, CD = 7 cm, BC = 3 cm, then length of AD is

(a) 7 cm

(b) 2 cm

(c) 8 cm

(d) none of these

Ans. (c) 8 cm

18. A tangent PQ at point P of a circle of radius 12 cm meets a line through the centre O to a point Q so that OQ = 20 cm, thenlength of PQ is

(a) 14 cm

(b) 15 cm

(c) 16 cm

(d) 10 cm

Ans. (d) 10 cm

19. A line intersecting a circle in two points is called

(a) tangent

(b) secant

(c) diameter

(d) none of these

Ans. (b) secant

20. The length of tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. What will be the radius of circle?

(a) 1 cm

(b) 2 cm

(c) 3 cm

(d) none of these

Ans. (c) 3 cm

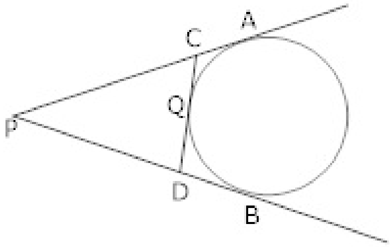
21. In the figure given below, PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PB = 12 cm and CQ = 3 cm, what is the length of PC?

(a) 9 cm

(b) 10 cm

(c) 1 cm

(d) 13 cm



Ans. (a) 9 cm

22. The tangent of a circle makes angle with radius at point of contact

(a) 60°

(b) 30°

(c) 90°

(d) none of these

Ans. (c) 90°

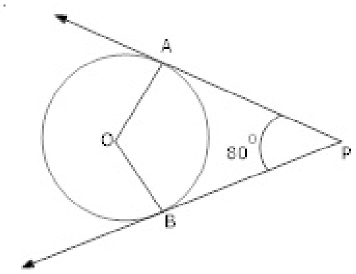
23. If tangent PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then what is the value of $\angle POA$?

(a) 30°

(b) 50°

(c) 70°

(d) 90°



Ans. (b) 50°

CBSE Class 10 Mathematics

Important Questions

Chapter 10

Circles

2 Marks Questions

1. Fill in the blanks:

(i) A tangent to a circle intersects it in ______________ point(s).

(ii) A line intersecting a circle in two points is called a ______________.

(iii) A circle can have ______________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ______________.

Ans. (i) A tangent to a circle intersects it in one point.

(ii) A line intersecting a circle in two points is called a secant.

(iii) A circle can have two parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called point of contact.

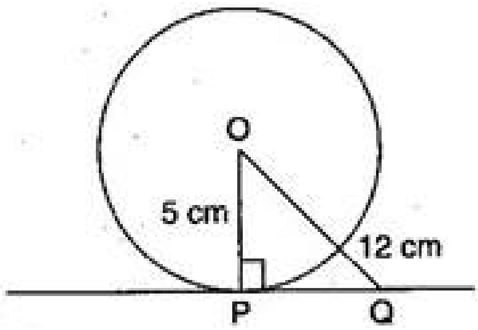
2. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ **cm**



Ans. (D) ∵ PQ is the tangent and OP is the radius through the point of contact.

∴ $\angle OPQ = 90^\circ$ [The tangent at any point of a circle is ⊥ to the radius through the point of contact]

∴ In right triangle OPQ,

$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$\Rightarrow (12)^2 = (5)^2 + PQ^2$

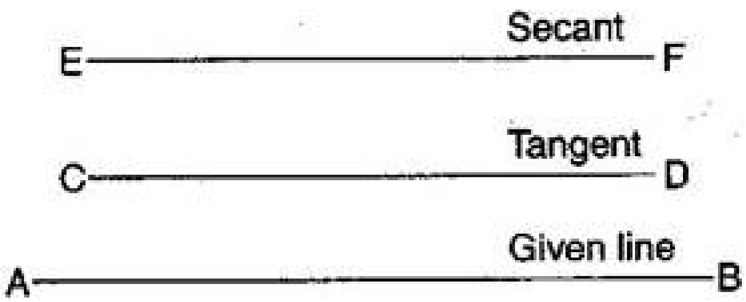
$\Rightarrow 144 = 25 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

3. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans.



4. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

(A) 7 cm

(B) 12 cm

(C) 15 cm

(D) 24.5 cm

Ans. (A) $\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\therefore$ In right triangle OPQ,

$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

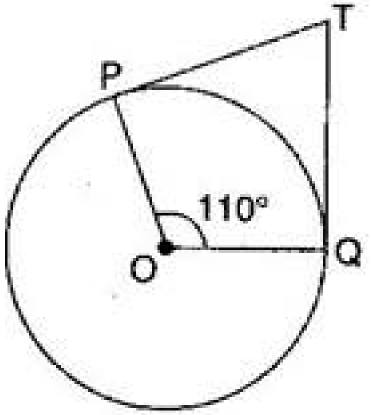
$\Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow 625 = OP^2 + 576$

$\Rightarrow OP^2 = 625 - 576 = 49$

$\Rightarrow OP = 7$ cm

5. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle$ POQ = 110°, **then $\angle$ PTQ is equal to:**



(A) 60°

(B) 70°

(C) 80°

(D) 90°

Ans. (B) $\angle$ POQ = 110°, $\angle$ OPT = 90° and $\angle$ OQT = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In quadrilateral OPTQ,

$\angle$ POQ + $\angle$ OPT + $\angle$ OQT + $\angle$ PTQ = 360°

[Angle sum property of quadrilateral]

$\Rightarrow 110^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow 290^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 70^\circ$

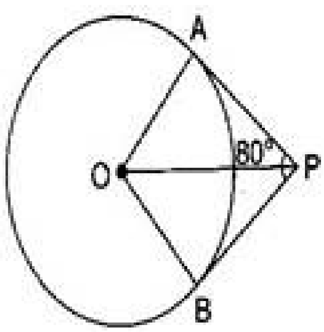
6. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to:

(A) 50°

(B) 60°

(C) 70°

(D) 80°



Ans. (A) $\because$ $\angle$ OPQ = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\angle$ OPA = $\frac{1}{2}\angle$BPA[Centre lies on the bisector of the angle between the two tangents]

In Δ OPA,

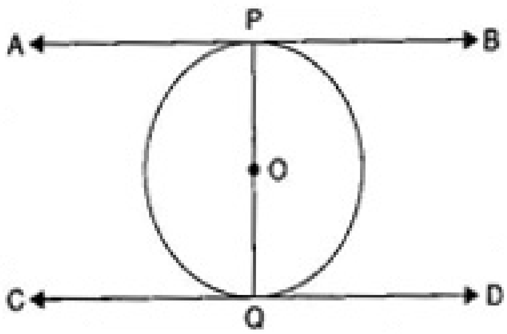
$\angle$ OAP + $\angle$ OPA + $\angle$ POA = 180°[Angle sum property of a triangle]

$\Rightarrow 90^\circ + 40^\circ$ + $\angle$ POA = 180°

$\Rightarrow 130^\circ$ + $\angle$ POA = 180°

$\Rightarrow \angle$ POA = 50°

7. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Ans. Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove: AB $||$ CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore$ $\angle$ OPA = 90° ....(i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore$ $\angle$ OQD = 90° ....(ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle OPA = \angle OQD$

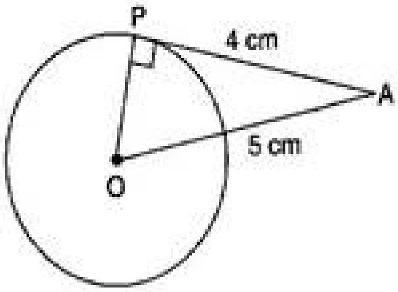
But these form a pair of equal alternate angles also,

$\therefore AB \parallel CD$

8. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans. We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact and the radius essentially passes through the centre of the circle, therefore the perpendicular at the point of contact to the tangent to a circle passes through the centre.

9. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.



Ans. We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.

$\therefore \angle OPA = 90^\circ$

$\therefore OA^2 = OP^2 + AP^2$ [By Pythagoras theorem]

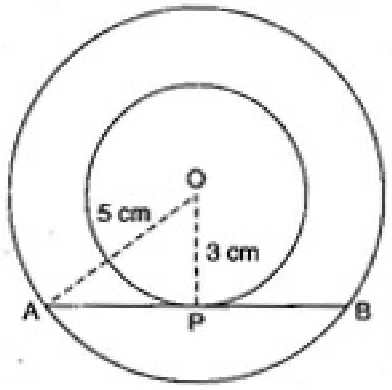
$\Rightarrow (5)^2 = (OP)^2 + (4)^2$

$\Rightarrow 25 = (OP)^2 + 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3$ cm

10. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.



Ans. Let O be the common centre of the two concentric circles.

Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle$ OPA = 90°

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact

$\therefore$ $OA^2 = OP^2 + AP^2$ [By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = 4 cm

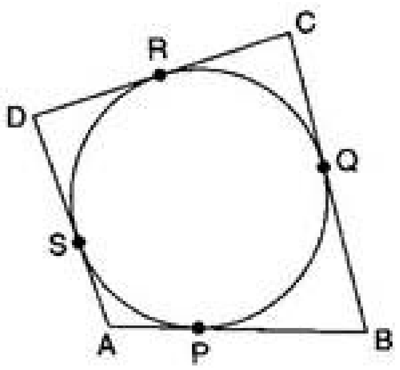
Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

AP = BP = 4 cm

$\Rightarrow$ AB = AP + BP = AP + AP = 2AP = 2 x 4 = 8 cm

11. A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

AB + CD = AD + BC



Ans. We know that the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

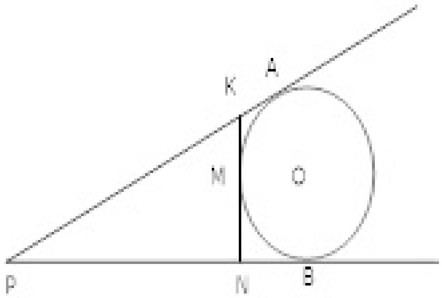
12. In two concentric circles prove that all chords of the outer circle which touch the inner circle are of equal length.

Ans. AB and CD are two chords of the circle which touch the inner circle at M and N.

Respectively $\therefore OM = ON$

$\Rightarrow AB = CD$ [$\because$ AB and CD are two chords of larger circle]

13. PA and PB are tangents from P to the circle with centre O. At the point M, a tangent is drawn cutting PA at K and PB at N. Prove that KN=AK+BN.



Ans. We know that the lengths of the tangents drawn form an external point to a circle are equal.

$\therefore PA = PB$.........(*i*)

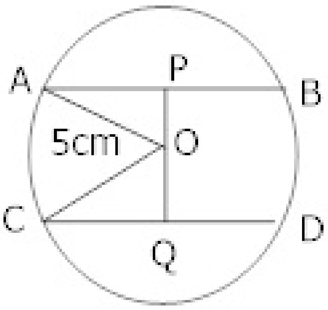
$KA = KM$......(*ii*)

$NB = NM$.......(*iii*)

(ii) + (iii)

$$KA + NB = KM + NM$$
$$\Rightarrow AK + BN = KM + MN$$
$$\Rightarrow AK + BN = KN$$

14. In the given figure, O is the centre of the circle with radius 5 cm and AB||CD. If AB = 6 cm, find OP.



Ans. $\because OP \perp AB$

$\therefore$ OP bisects AB

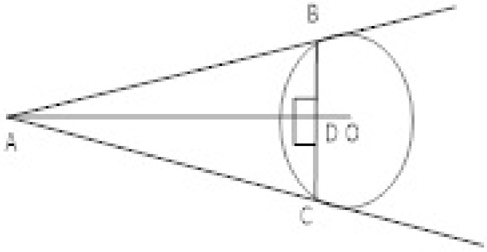
$$\therefore AP = \frac{1}{2}AB = \frac{1}{2} \times 6 = 3\ cm$$

From right ΔOAP, $OA^2 = OP^2 + AP^2$

$\Rightarrow 5^2 = OP^2 + 3^2$

$\Rightarrow OP = 4\ cm$

15. Prove that the tangents at the end of a chord of a circle make equal angles with the chord.



Ans. In ΔADB and ΔADC,

BD = DC

And $\angle ADB = \angle ADC = 90°$

AD = AD [Common]

$\therefore \Delta ADB \cong \Delta ADC$ [SAS]

$\therefore \angle ABD = \angle ACD$ [By CPCT]

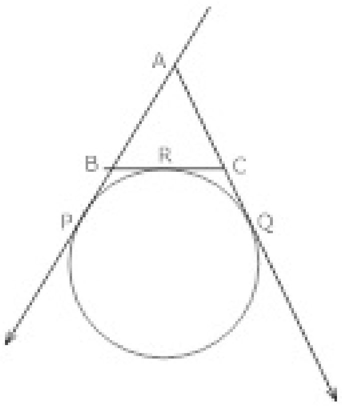
16. Find the locus of the centre of circles which touch a given line at a given point.

Ans. Let APB be the given line and let a circle with centre O touch APB at P. Then $\angle OPB = 90°$, let there be another circle with centre O' which touches the line APB at P.

Thus, $\angle O'PB = 90°$

$\therefore \angle OPB = \angle O'PB = 90°$

17. In the given figure, find the perimeter of ΔABC, if AP = 10 cm.



Ans. $\because$ BC touches the circle at R

$\because$ Tangents drawn from external point to the circle are equal.

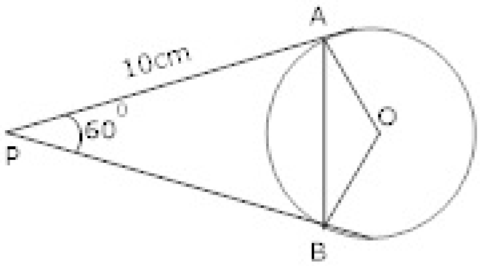
$\therefore$ AP = AQ, BR = BP

And CR = CQ

$\therefore$ Perimeter of $\Delta ABC = AB + BC + AC$

$= AB + (BR + RC) + AC$
$= AB + BP + CQ + AC$
$= AP + AQ = 2AP = 2 \times 10 = 20cm$

18. If PA and PB are tangents drawn from external point P such that PA = 10cm and $\angle APB = 60°$, find the length of chord AB.



Ans. $\because \angle APB = 60°$

$\angle AOB = 120°$ [O is centre of circle]

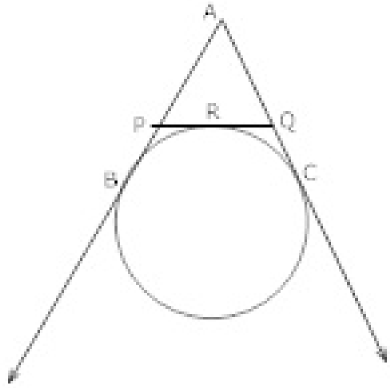
$\angle OAB = \angle OBA = 30°$

$\therefore \angle PAB = 60^\circ, \angle PBA = 60^\circ$

$\therefore \Delta PAB$ is equilateral triangle

$\therefore AB = PA = 10cm$

19. If AB, AC and PQ are tangents in the given figure and AB = 25cm, find the perimeter of ΔAPQ.



Ans. Perimeter of $\Delta APQ = AP + AQ + PQ$

$= AP + AQ + PX + XQ$

$= (AP + PB) + (AQ + QC)$

$= AB + AC$

$= 2AB = 2 \times 25 = 50cm$

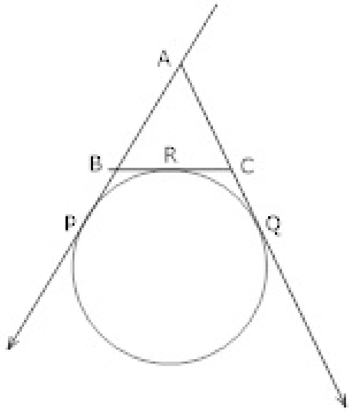
20. Find the locus of the centre of circles which touch a given line at a given point.

Ans. Let APB be the given line and let a circle with centre O touch APB at P. Then $\angle OPB = 90^\circ$, let there be another circle with centre O' which touches the line APB at P.

Thus, $\angle O'PB = 90^\circ$

$\therefore \angle OPB = \angle O'PB = 90^\circ$

21. In the given figure, find the perimeter of ΔABC, if AP = 10 cm.



Ans. $\because$ BC touches the circle at R

$\because$ Tangents drawn from external point to the circle are equal.

$\therefore$ AP = AQ, BR = BP

And CR = CQ

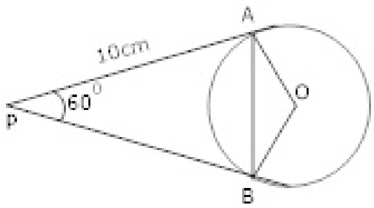
$\therefore$ Perimeter of $\Delta ABC = AB + BC + AC$

$= AB + (BR + RC) + AC$

$= AB + BP + CQ + AC$

$= AP + AQ = 2AP = 2 \times 10 = 20cm$

22. If PA and PB are tangents drawn from external point P such that PA = 10 cm and $\angle APB = 60°$, find the length of chord AB.



Ans. $\because \angle APB = 60°$

$\angle AOB = 120°$ [O is centre of circle]

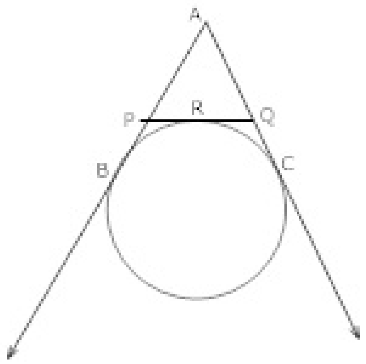
$\angle OAB = \angle OBA = 30°$

$\therefore \angle PAB = 60°, \angle PBA = 60°$

$\therefore \Delta PAB$ is equilateral triangle

$\therefore AB = PA = 10cm$

23. If AB, AC and PQ are tangents in the given figure and AB = 25cm, find the perimeter of ΔAPQ.



Ans. Perimeter of $\Delta APQ = AP + AQ + PQ$

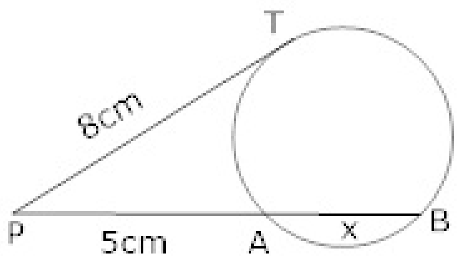
$= AP + AQ + PX + XQ$

$= (AP + PB) + (AQ + QC)$

$= AB + AC$

$= 2AB = 2 \times 25 = 50cm$

24. Find the unknown length x.



Ans. $\because$ PT is tangent to a circle and PAB is a secant.

$\therefore PA.PB = PT^2$

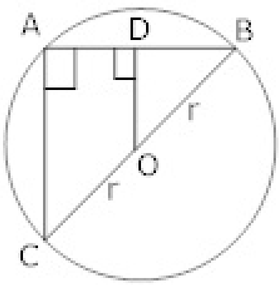
$\Rightarrow 5(5 + x) = 8^2$

$\Rightarrow 25 + 5x = 64$

$\Rightarrow x = \frac{39}{8} = 7.8cm$

25. In the given figure, OD is perpendicular to the chord AB of a circle whose centre is

O. If BC is a diameter, find $\frac{CA}{OD}$.



Ans. Since BC is a diameter

$\therefore \angle CAB = 90°$

Also OD $\perp$ AB

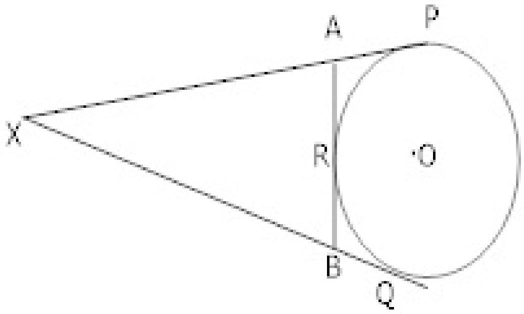
$\therefore \angle ODB = 90°$ $\Delta ACB \sim \Delta DOB$

$[\because \angle CAB = \angle ODB = 90°]$

$\angle ABC = \angle DBO$ [Common]

$\therefore \frac{CA}{OD} = \frac{CB}{OB} = \frac{2r}{r} = 2$

26. In the given figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle such that ARB is a tangent to the circle prove that XA + AR = XB + BR.



Ans. In the given figure, XP and XQ are tangents from external point

$\therefore XP = XQ$........(i)

$AR = AP$...........(ii)

$BR = BQ$...........(iii)

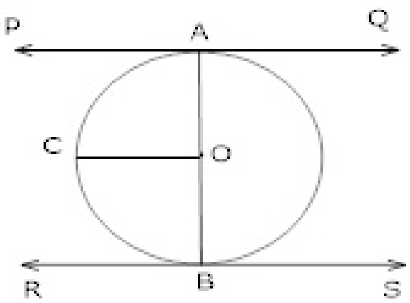
[∵ Length of tangents are equal from external point]

$XP = XQ$

$XA + AP = XB + BQ$ [By (ii) and (iii)]

$XA + AR = XB + BR$ [By (ii) and (iii)]

27. Prove that the segment joining the points of contact of two parallel tangents, passes through the centre.

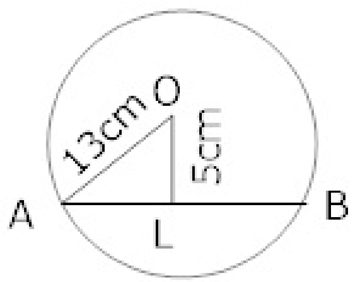


Ans. Given two parallel tangents PQ and RS of a circle with centre O

Draw line OC||RS.

i.e., $\angle PAO + \angle COA = 180°$

28. In figure, if OL = 5 cm, OA = 13 cm, then length of AB is



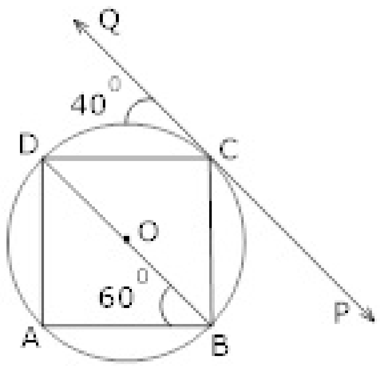
Ans. $AB = 2AL = 2\sqrt{OA^2 - OL^2}$

$= 2\sqrt{13^2 - 5^2}$

$= 2\sqrt{169 - 25} = 2\sqrt{144}$

$= 2 \times 12 = 24cm$

29. In the given figure, ABCD is a cyclic quadrilateral and PQ is a tangent to the circle at C. If BD is a diameter, $\angle OCQ = 40^\circ$ and $\angle ABD = 60^\circ$, find $\angle BCP$.

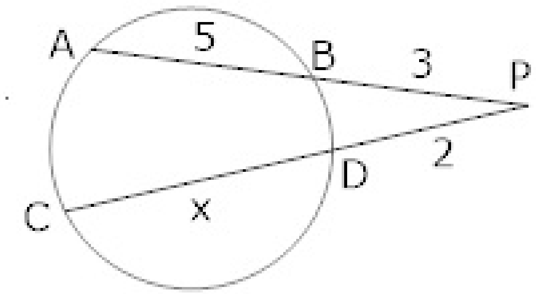


Ans. $\because$ BD is a diameter

$\therefore \angle BCD = 90^\circ$ [Angle in the semi-circle]

$\therefore \angle BCP = 180^\circ - 90^\circ - 40^\circ = 50^\circ$

30. Two chords AB and CD of a circle intersect each other at P outside the circle. If AB = 5 cm, BP = 3 cm and PD = 2cm, find CD.



Ans. $\because$ Two chords AB and CD of a circle intersect each other at P

$\therefore$ PA × PB = PC × PD [length of tangent from P]

$\Rightarrow (AB + PB) \times PB = (PD + PC) PD$

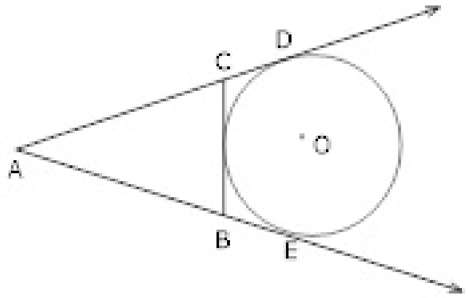
$\Rightarrow (5+3)(3) = (2+x)^2$

$\Rightarrow 24 = (2+x)^2$

$\Rightarrow x = 10 \Rightarrow CD = 10\ cm$

31. In the adjoining figure, if AD, AE and BC are tangents to the circle at D, E and F

respectively, then prove that 2 AD=AB+BC+CA.



Ans. CD = CF, BE = BF

$\Rightarrow CD + BE = CF + BF = BC$

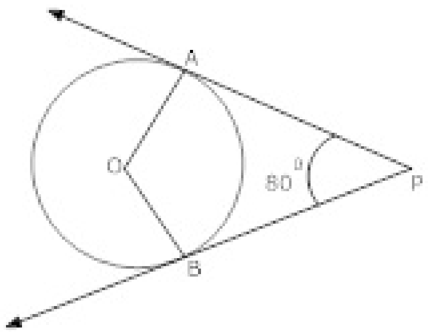
$Now\ \ AD = AC + CD = AC + CF$

$AE = AB + BE = AB + BF$

$\therefore AD + AE = AB + AC + BC$

$\Rightarrow 2AD = AB + BC + AC$

32. In figure, PA and PB are tangents from P to the circle with centre O. R is a point on the circle,prove that PC + CR = PD + DR.



Ans. Since length of tangents from an external point to a circle are equal in length

$\therefore PA = PB$

$CA = CR$ ...(i)

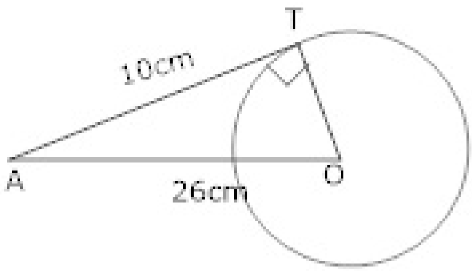
And DB = DR

Now PA = PB

$\Rightarrow$ PC + CA = PD + DB

$\Rightarrow$ PC + CR = PD + DR [By (i)]

33. The length of tangents from a point A at distance of 26 cm from the centre of the circle is 10cm, what will be the radius of the circle?



Ans. Since tangents to a circle is perpendicular to radius through the point of contact

$\therefore \angle OTA = 90°$

In right $\Delta OTA = 90°$, we have

$OA^2 = OT^2 + AT^2$

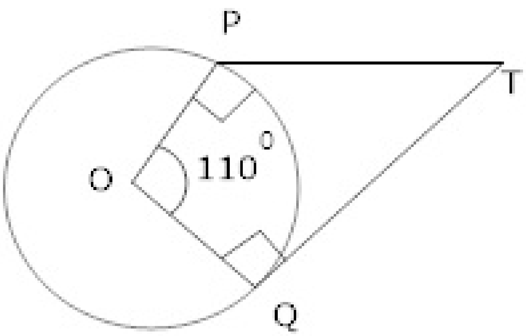
$\Rightarrow (26)^2 = OT^2 + (10)^2$

$\Rightarrow OT^2 = 676 - 100$

$\Rightarrow OT^2 = 576$

$\Rightarrow OT = 24$

34. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTO$.

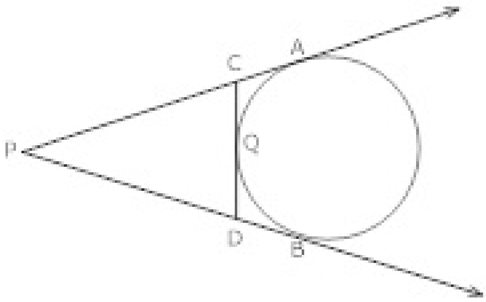


Ans. Since $\angle POQ + \angle PTO = 180° [\because \angle OPT = 90°, \angle OQT = 90°]$

$\Rightarrow 110° + \angle PTQ = 180°$

$\Rightarrow \angle PTQ = 180° - 110° = 70°$

35. In the figure, given below PA and PB are tangents to the circle drawn from an external point P. CD is thethird tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length of PC?



Ans. PA=PB=10 cm

CQ = CA = 2 cm

PC = PA – CA = 10 – 2 = 8 cm

CBSE Class 10 Mathematics

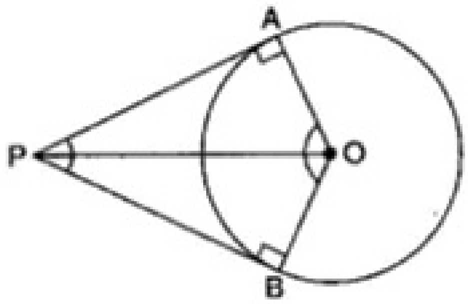
Important Questions

Chapter 10

Circles

3 Marks Questions

1. Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



Ans. $\angle OPA = 90^\circ$ ....(i)

$\angle OCA = 90^\circ$ ....(ii)

[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^\circ$

[Angle sum property of a quadrilateral]

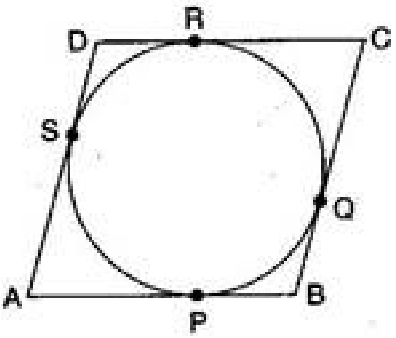
$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$

[From eq. (i) & (ii)]

$\Rightarrow \angle APB + \angle AOB = 180^\circ$

$\therefore \angle APB$ and $\angle AOB$ are supplementary.

2. Prove that the parallelogram circumscribing a circle is a rhombus.



Ans. Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore$ AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

(AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS)

$\Rightarrow$ AB + CD = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD [Opposite sides of $\|$gm are equal]

$\Rightarrow$ 2AB = 2AD

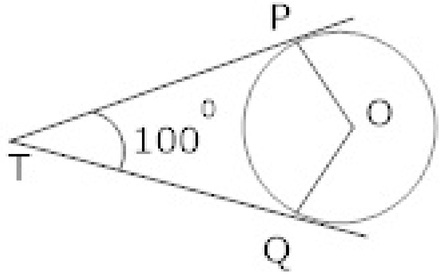
$\Rightarrow$ AB = AD

But AB = CD and AD = BC [Opposite sides of $\|$gm]

$\therefore$ AB = BC = CD = AD

$\therefore$ Parallelogram ABCD is a rhombus.

3. Two tangents TP and TQ are drawn from an external point T with centre O as shown in figure. If they are inclined to each other at an angle of 100^0, then what is the value of $\angle POQ$?



Ans. $\because TP$ and TQ are tangents and O is the centre of the circle

$\therefore OP \perp PT, OQ \perp QT$

$\therefore \angle TPO + \angle TQO = 180°$

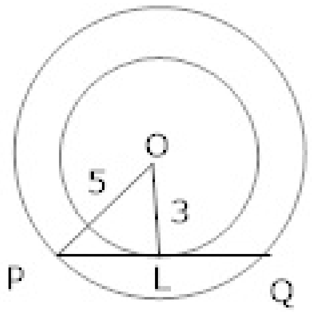
$\therefore$ Quadrilateral OPTQ is cyclic.

$\therefore \angle PTQ + \angle POQ = 180°$

$\therefore 100° + \angle POQ = 180°$

$\therefore \angle POQ = 180° - 100° = 80°$

4. Two concentric circles are of radii 5 cm and 3 cm,find the length of the chord of the larger circle which touches the smaller circle.



Ans. $\because$ PQ is the chord of the larger circle which touches the smaller circle at the point L. Since PQ is tangent at the point L to the smaller circle with centre O.

$\therefore OL \perp PQ$

$\because PQ$ is a chord of the bigger circle and $OL \perp PQ$

$\therefore OL$ bisects PQ

$\therefore PQ = 2PL$

In $\Delta OPL,\quad PL = \sqrt{OP^2 - OL^2}$

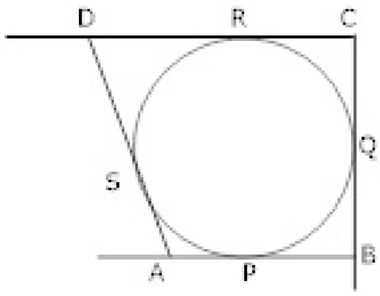
$= \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9} = 4$

$\therefore$ Chord PQ = 2PL =8 cm

$\therefore$ Length of chord PQ = 8 cm

5. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB+CD=AD+BC.



Ans. $\because$ AP, AS are tangents from a point A (Outside the circle) to the circle.

$\therefore AP = AS$

Similarly, BP = BQ

CQ = CR

DR = DS

Now AB + CD = AP + PB + CR + RD

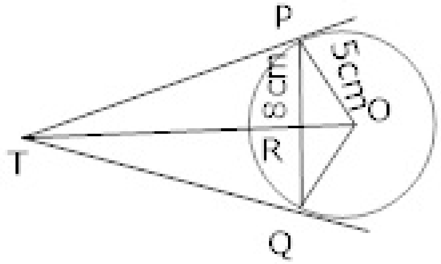
= AS +BQ + CQ +DS

= (AS +DS) + (BQ + CQ)

= AD + BC

\AB + CD = AD + BC

6. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at point T. Find the length TP.



Ans. Join OT.

TP = PQ [tangents from T upon the circle]

$\therefore OT \perp PQ$

And OT bisects PQ

$\therefore PR = PQ = 4cm$

Now $OR = \sqrt{OP^2 - PR^2}$

$OR = \sqrt{5^2 - 4^2} = 3cm$

Now $\angle TPR + \angle RPO = 90^\circ$ [$\because \angle TPO=90^\circ$]

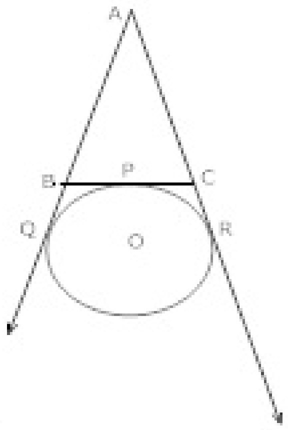
$= \angle TPR + \angle PTR$

$\therefore \angle RPO = \angle PTR$

$\Delta TRP \sim \Delta TRQ$ [By AA similarity]

$$\therefore \frac{TP}{PO} = \frac{RP}{RO}$$
$$\Rightarrow \frac{TP}{5} = \frac{4}{3}$$
$$\Rightarrow TP = \frac{20}{3} cm$$

7. A circle is touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that AQ = $\frac{1}{2}$ (perimeter of ΔABC).



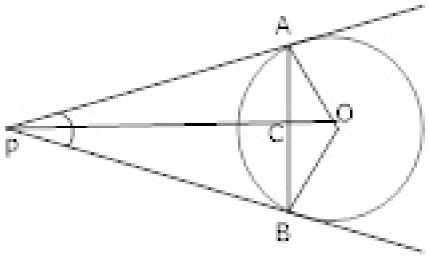
Ans. We know that the two tangents drawn to a circle from an external point are equal.

$$\therefore AQ = AR,\ BP = BQ,\ CP = CR$$

$\therefore$ Perimeter of $\Delta ABC = AB + BC + AC$

$$= AB + BP + PC + AC$$
$$= AB + BQ + CR + AC \quad [\because BP = BQ, PC = CQ]$$
$$= AQ + AR = 2AQ = 2AR \quad [\because AQ = AR]$$
$$= AQ = AR = \frac{1}{2} \text{ [perimeter of } \Delta ABC]$$

8. If PA and PB are two tangents drawn from a point P to a circle with centreO touching it at A and B. Prove that OP is the perpendicular bisector of AB.



Ans. Let OP intersect AB at a point C, we have to prove that AC = CB and $\angle ACP = \angle BCP = 90°$

$\because PA, PB$ are two tangents from a point P to the circle with centre O

$\therefore \angle APO = \angle BPO$ [$\because$ O lies on the bisector of $\angle APB$]

In two ΔS , ACP and BCP, we have

AP = BP [$\because$ tangents from P to the circle are equal]

PC = PC [Common]

$\angle APO = \angle BPO$ [Proved]

$\therefore \Delta ACP \cong \Delta BCP$ [By SAS rule]

$\therefore AC = CB$ [CPCT]

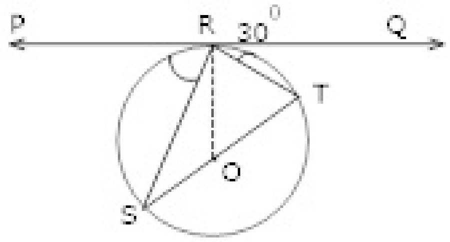
And $\angle ACP = \angle BCP [CPCT]$

But $\angle ACP + \angle BCP = 180°$

$\Rightarrow \angle ACP = \angle BCP = 90°$

Hence, OP is perpendicular bisector of AB.

9. In the given figure, PQ is tangent at point R of the circle with centre O. If $\angle TRQ = 30°$, find $m\angle PRS$.



Ans. Given PQ is tangent at point R and $\angle TRQ = 30^\circ$

$\angle PRQ = 180^\circ$

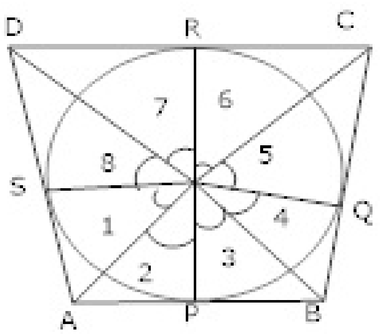
$\angle QRT = 30^\circ$

$\angle TRS = 90^\circ$ [∵ Tangent of a circle is perpendicular to Radius]

$\therefore \angle PRS = 180^\circ - 120^\circ = 60^\circ$

$\therefore \angle PRS = 180^\circ - 120^\circ = 60^\circ$

10. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Ans. Let the circle touch the sides AB, BC, CD and DA at the points P, Q, R, and S respectively.

Join OP, OQ, OR and OS.

Join OA, OB, OC and OD.

Since the two tangents drawn from an external point subtend equal angles at the centre.

$\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 + = 360^\circ$

[∵ Sum of all angles around a point = 360°]

$\therefore 2[\angle 2+\angle 3+\angle 6+\angle 7]=360^\circ$

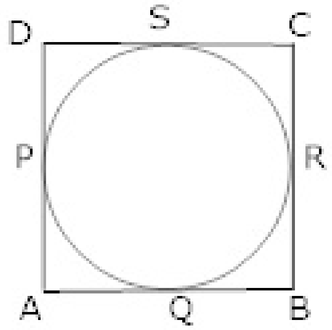
And $2(\angle 4+\angle 5+\angle 8+\angle 1)=360^\circ$

$\Rightarrow(\angle 2+\angle 3)+(\angle 6+\angle 7)=180^\circ$

And $(\angle 4+\angle 5)+(\angle 8+\angle 1)=180^\circ$

$\Rightarrow \angle AOB+\angle COD=180^\circ$ and $\angle BOC+\angle AOD=180^\circ$

11. Prove that parallelogram circumscribing a circle is a rhombus.



Ans. Given ABCD is a parallelogram in which all the sides touch a given circle

To prove: ABCD is a rhombus

Proof:

∵ ABCD is a parallelogram

∴ AB=DC and AD = BC

Again AP, AQ are tangents to the circle from the point A

$\therefore AP=AQ$

Similarly, $BR=BQ$

$CR=CS$

$DP=DS$

$\therefore (AP+DP)+(BR+CR)$
$= AQ+DS+BQ+CS$
$= (AQ+BQ)+(CS+DS)$

$\Rightarrow AD+BC = AB+DC$

$\Rightarrow BC+BC = AB+AB$
$[\because AB = DC, AD = BC]$

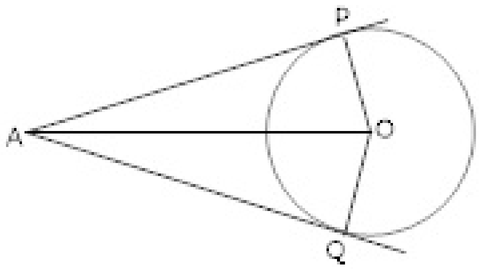
$\Rightarrow 2BC = 2AB$
$\Rightarrow BC = AB$

Hence, parallelogram ABCD is a rhombus.

12. If two tangents are drawn to a circle from an external point then

(i) they subtend equal angles at the centre.

(ii) they are equally inclined to the segment joining the centre to that point.



Ans. Given on a circle C (O,r), two tangents AP and AQ are drawn from an external point A.

To prove:

(i) $\angle AOP = \angle AOQ$

(ii) $\angle OAP = \angle OAP$

Construction: Join AO, PO and QO

Proof: In ΔAPQ and ΔAQO, AP = AQ [Length of the tangents drawn from an external point]

PO = QO [Radii of the same circle]

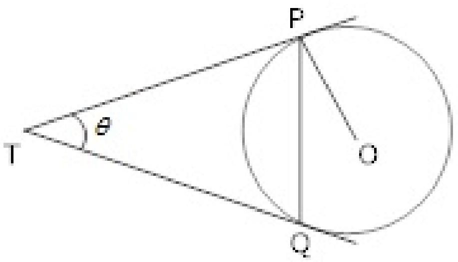
AO = AO [common]

$\Delta APO \cong \Delta AQO$ [By SSS theorem of congruence]

(i) $\angle AOP = \angle AOQ$ [CPCT]

(ii) $\angle OAP = \angle QAO$ [By CPCT.]

13. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Ans. Given A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove: $\angle PTQ = 2\angle OPQ$

Proof: Let $\angle PTQ = \theta$

Since TP, TQ are tangents drawn from point T to the circle.

TP = TQ

∴ TPQ is an isosceles triangle

$\therefore \angle TPQ = \angle TQP = \frac{1}{2}(180° - \theta)$

$= 90° - \frac{\theta}{2}$

Since, TP is a tangent to the circle at point of contact P

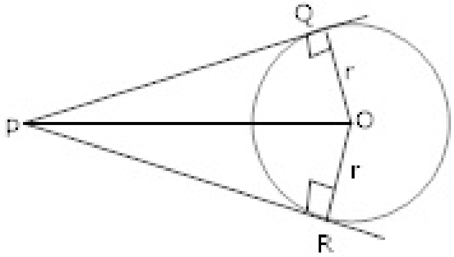
$\therefore \angle OPT = 90°$

$\therefore \angle OPQ = \angle OPT - \angle TPQ$

$= 90° - \left(90° - \frac{1}{2}\theta\right) = \frac{\theta}{2} = \frac{1}{2}\angle PTQ$

Thus, $\angle PTQ = 2\angle OPQ$

14. Prove that the lengths of two tangents drawn from an external point to a circle are equal.



Ans. Given: P is an external point to the circle C(O,r).

PQ and PR are two tangents from P to the circle.

To Prove: PQ = PR

Construction: Join OP

Proof:

∵ A tangent to a circle is perpendicular to the radius through the point of contact

$\therefore \angle OQP = 90° = \angle ORP$

Now in right triangles POQ and POR,

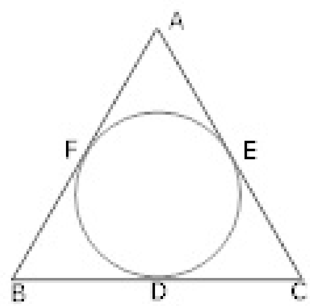
OQ = OR [Each radius r]

Hypotenuse. OP = Hypotenuse. OP [common]

$\therefore \Delta POQ \cong \Delta POR$ [By RHS rule]

$\therefore PQ = PR$

15. The circle of ΔABC touches the sides BC, CA and AB at D,E and F respectively. If AB = AC, prove that BD = CD.



Ans. $\because$ Tangents drawn from an external point to a circle are equal in length

$\therefore$ AF = AE [Tangents from A] ...(i)

BF = BD [Tangents from B] ...(ii)

CD = CE [Tangents from C] ...(iii)

Adding (i), (ii)and (iii), we get

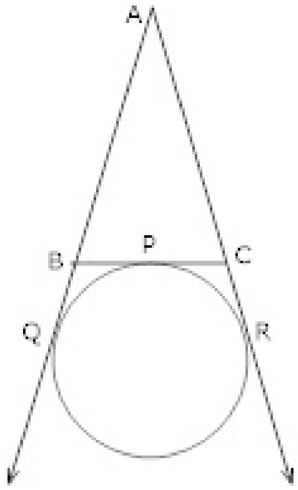
AF + BF + CD = AE + BD + CE

$\Rightarrow AB + CD = AC + BD$

But AB = AC (given)

CD = BD

16. A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced at Q and R respectively, show that AQ = $\frac{1}{2}$ [Perimeter of ΔABC].



Ans. Since the tangents from an external point to a circle are equal in length,

\BP = BQ ...(i) [from point B]

CP = CR ...(ii) [from point C]

And AQ = AR ...(iii) [From point A]

From (iii), we have

AQ = AR

$\Rightarrow AB + BQ = AC + CR$

$\Rightarrow AB + BP = AC + CP ....(iv)$ [Using (i) and (ii)]

Now perimeter of ΔABC

AB + BC +AC = AB +(BP + PC) + AC

= (AB + BP) + (AC +PC)

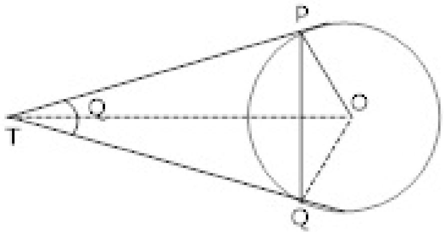
= 2 (AB + BP) [using (iv)]

=2 (AB + BQ) [using (i)]

= 2 AQ

ÞAQ = $\frac{1}{2}$ (perimeter of ΔABC)

17. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Ans. Given: A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P and Q are the points of contact.

To prove: $\angle PTQ = 2\angle OPQ$

Proof: Let $\angle PTQ = \theta$

In ΔTPQ, we have

TP = TQ

[Length of the tangents drawn from an external point to a circle are equal]

So, TPQ is an isosceles triangle.

$\therefore \angle TPQ = \angle TOP$..........$(i)$

In ΔTPQ, we have

$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$ [$\because$ Sum of three angles of a Δ is 180°]

$\Rightarrow 2\angle TPQ + Q = 180^\circ$........$(i)$

$\Rightarrow 2\angle TPQ = 180^\circ - \theta$

$\Rightarrow \angle TPQ = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta$........$(ii)$

But $\angle OPT = 90^\circ$........(iii)

[Angle between the tangent and radius of a circle is 90°]

Now $\angle OPQ = \angle OPT - \angle TPQ$

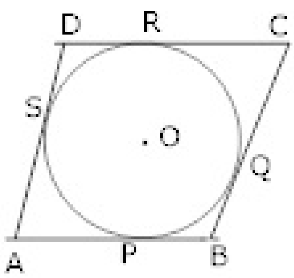
$$= 90° - \left[90° - \frac{1}{2}\theta\right]$$

$$= \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$$

$$\Rightarrow \angle OPQ = \frac{1}{2}\angle PTQ$$

$$\Rightarrow \angle PTQ = 2\angle OPQ$$

18. Prove that the parallelogram circumscribing a circle is a rhombus



Ans. Given: ABCD be the parallelogram circumscribing a circle with centre O such that the sides AB, BC, CD and DA touch a circle at P, Q, R and S respectively.

To prove: ||gm ABCD is a rhombus.

Proof: AP = AS ...(i)

BP = BQ ...(ii)

CR = CQ ...(iii)

DR = DS ...(iv)

[Tangents drawn from an external point to a circle are equal]

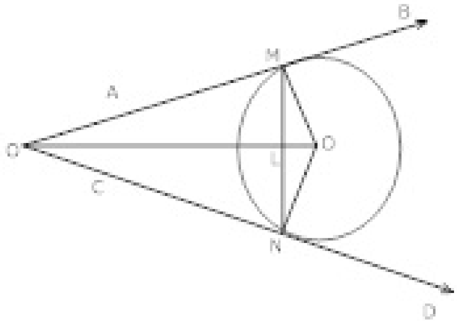
Adding (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

19. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with chord.



Ans. Let NM be chord of circle with centre C.

Let tangents at M.N meet at the point O.

Since OM is a tangent

$\therefore OM \perp CM$ i.e. $\angle OMC = 90°$

$\because ON$ is a tangent

$\therefore ON \perp CN$ i.e. $\angle ONC = 90°$

Again in $\Delta CMN, CM = CN = r$

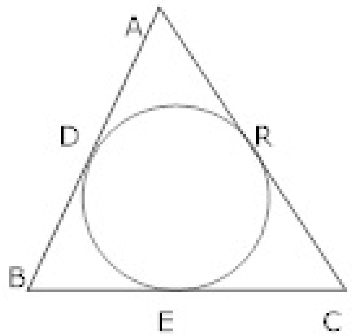
$\therefore \angle CMN = \angle CNM$

$\therefore \angle OMC - \angle CMN = \angle ONC - \angle CNM$

$\Rightarrow \angle OML = \angle ONL$

Thus, tangents make equal angle with the chord.

20. In the given figure, if AB = AC, prove that BE = EC.



Ans. Since tangents from an exterior point A to a circle are equal in length

$\therefore AD = AF$............(i)

Similarly, tangents from an exterior point B to a circle are equal in length

$\therefore BD = BE$............(2)

Similarly, for C

CE = CF ....(3)

Now AB = AC

$\therefore AB - AD = AC - AD$

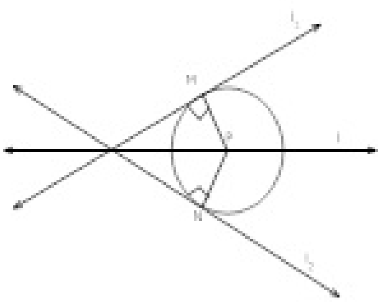
$\Rightarrow AB - AD = AC - AF$.........[By (i)]

$\Rightarrow BD = CF$

$\Rightarrow BE = CF$.........[By (ii)]

$\Rightarrow BE = CE$ $[\because BD = BE, CE = CF]$ [By (iii)]

21. Find the locus of centre of circle with two intersecting lines.



Ans. Let l_1, l_2 be two intersection lines.

Let a circle with centre P touch the two lines l_1 and l_2 at M and N respectively.

PM = PN [Radii of same circle]

$\therefore$ P is equidistance from the lines l_1 and l_2

Similarly, centre of any other circle which touch the two intersecting lines l_1, l_2 will be equidistant from l_1 and l_2

$\therefore$ P lies on l a bisector of the angle between l_1 and l_2

[$\because$ The locus of points equidistant from two intersecting lines is the pair of bisectors of the angle between the lines]

Hence, locus of centre of circles which touch two intersecting lines is the pair of bisectors of the angles between the two lines.

22. In the given figure, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90°$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius of the circle.

Ans. In the given figure, $OP \perp BC$ and OQ ^ BA

Also, OP = OQ =r

$\therefore OPBQ$ is a square

$\therefore$ BP = BQ =r

But DR = DS = 5 cm ...(i)

$$\therefore AR = AD - DR$$
$$= 23 - 5 = 18\ cm$$
$$AQ = AR = 18\ cm$$
$$BQ = AB - AQ$$
$$= 20 - 18 = 11\ cm$$
$$r = 11\ cm$$

CBSE Class 10 Mathematics

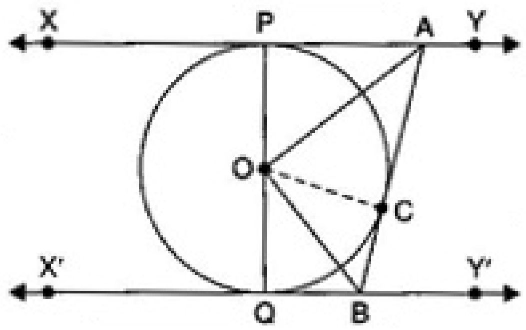
Important Questions

Chapter 10

Circles

4 Marks Questions

1. In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle$ AOB = 90°.



Ans. Given: In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove: $\angle$ AOB = 90°

Construction: Join OC

Proof: $\angle$ OPA = 90° ....(i)

$\angle$ OCA = 90° ....(ii)

[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In right angled triangles OPA and OCA,

OA = OA [Common]

AP = AC [Tangents from an external point to a circle are equal]

$\therefore$ $\triangle$OPA $\cong$ $\triangle$OCA [RHS congruence criterion]

$\therefore \angle OAP = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \angle OAC = \frac{1}{2}\angle PAB$ ....(iii)

Similarly, $\angle OBQ = \angle OBC$

$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$ ....(iv)

$\because$ XY || X'Y' and a transversal AB intersects them.

$\therefore \angle PAB + \angle QBA = 180^\circ$ [Sum of the consecutive interior angles on the same side of the transversal is 180°]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA = \frac{1}{2}\times 180^\circ$ ....(v)

$\Rightarrow \angle OAC + \angle OBC = 90^\circ$

[From eq. (iii) & (iv)]

In ΔAOB,

$\angle OAC + \angle OBC + \angle AOB = 180^\circ$

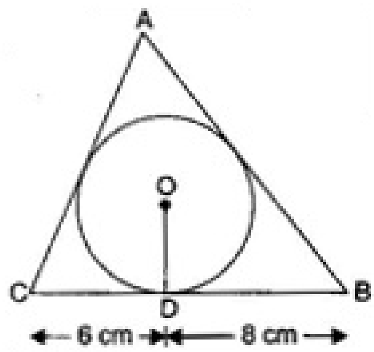
[Angel sum property of a triangle]

$\Rightarrow 90^\circ + \angle AOB = 180^\circ$

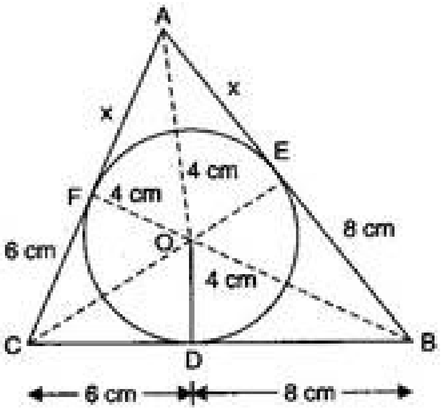
[From eq. (v)]

$\Rightarrow \angle AOB = 90^\circ$

2. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Ans. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

$\therefore$ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

$\therefore$ Semi-perimeter of ΔABC

$= \dfrac{(x+6)+(x+8)+(6+8)}{2}$

$= (x+14)$ cm

$\therefore$ Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

$= \sqrt{(x+14)(x)(6)(8)}$ cm^2

Now, Area of $\triangle$OBC = Area of $\triangle$OBC + Area of $\triangle$OCA + Area of $\triangle$OAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= 28+2x+12+2x+16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= 4x+56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14)$

Squaring both sides,

$(x+14)(x)(6)(8) = 16(x+14)^2$

$\Rightarrow 3x = x+14$

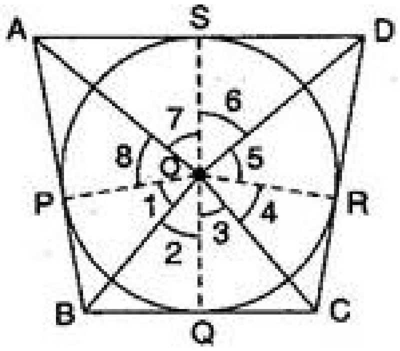
$\Rightarrow 2x = 14$

$\Rightarrow x = 7$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

And AC = $x+6$ = 7 + 6 = 13 cm

3. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Ans. Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) $\angle$ AOB + $\angle$ COD = 180° (ii) $\angle$ BOC + $\angle$ AOD = 180°

Construction: Join OP, OQ, OR and OS.

Proof: Since tangents from an external point to a circle are equal.

$\therefore$ AP = AS,

BP = BQ ....(i)

CQ = CR

DR = DS

In $\triangle$OBP and $\triangle$OBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

$\therefore$ $\triangle$OPB $\cong$ $\triangle$OBQ [By SSS congruence criterion]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360°$.

$\therefore\ \angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360°$

$\Rightarrow\ \angle 1+\angle 1+\angle 4+\angle 4+\angle 5+\angle 5+\angle 8+\angle 8=360°$

$\Rightarrow\ 2(\angle 1+\angle 4+\angle 5+\angle 8)=360°$

$\Rightarrow\ \angle 1+\angle 4+\angle 5+\angle 8=180°$

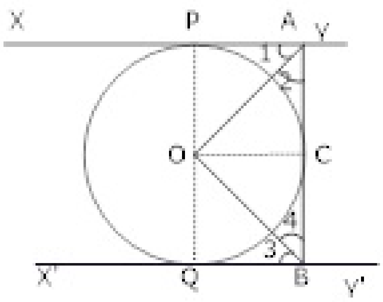
$\Rightarrow\ (\angle 1+\angle 5)+(\angle 4+\angle 8)=180°$

$\Rightarrow\ \angle$ AOB + $\angle$ COD = $180°$

Similarly, we can prove that

$\angle$ BOC + $\angle$ AOD = $180°$

4. In the given figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB=90°$.



Ans. Join OC.

In ΔOAP and DAOC, we have

AP = AC [∵ tangents from A to the circle are equal]

AO = AO

OP = OC [radius]

$\therefore\ \Delta OAP\cong\Delta AOC$ [By CPCT]

$\therefore \angle 1 = \angle 2$

$\therefore \angle PAC = 2\angle 2$

Similarly, $\angle CBQ = 2\angle 4$

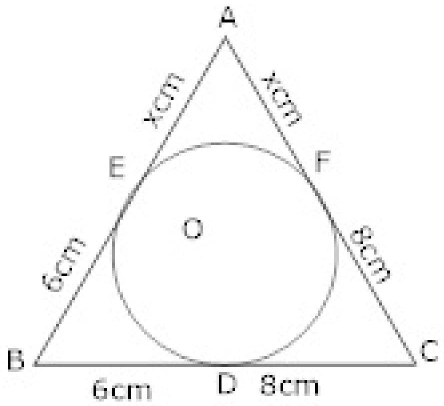
Now, $\angle PAC + \angle CBQ = 180^\circ$ $[\because XY \parallel X'Y']$

$2\angle 2 + 2\angle 4 = 180^\circ$
$\Rightarrow \angle 2 + \angle 4 = 90^\circ$

But in ΔAOB,

$\Rightarrow \angle AOB + \angle OAB + \angle ABO = 180^\circ$
$\Rightarrow \angle AOB + \angle 2 + \angle 4 = 180^\circ$
$\Rightarrow \angle AOB + 90^\circ = 180^\circ$
$\Rightarrow \angle AOB = 90^\circ$

5. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



Ans. Let the sides BC, CA, AB of ΔABC touch the incircle at D, E, F respectively.

Join the centre O of the circle with A, B, C, D, E, F

Since, tangents to a circle from an external point are equal

$\therefore CE = CD = 6cm$

$BF = BD = 8cm$

$AE = AF = xcm \quad (say)$

$OE = OF = OD = 4cm$ [Radii of the circle]

Area of $\Delta OAB = \frac{1}{2}(8+x)\times 4$

$= (16+2x)cm^2$.....(i)

Area of $\Delta OBC = \frac{1}{2}\times 14\times 4 = 28cm^2$.......$(ii)$

area $\Delta OCA = \frac{1}{2}(6+x)4 = 12+2x$........$(iii)$

$\therefore$ area $\Delta ABC = 16+2x+12+2x+28 = (4x+56)cm^2$........$(iv)$

Again, perimeter of $\Delta ABC = AC + AB + BC$

$= 6+x+(8+x)+(6+8)$

$= 28+2x = 2(14+x)cm$

$S = \frac{2(14+x)}{2} = 14+x$

Area of $\Delta AOC = \sqrt{S(s-a)(s-b)(s-c)}$

$= \sqrt{(14+x)(14+x-14)(14+x-6-x)(14+x-8-x)}$

$= \sqrt{(14+x)48x}$

$= \sqrt{672x+48x^2}$..........(v)

$\therefore (4x+56) = \sqrt{672x+48x2}$ [By 4 and 5]

$\Rightarrow (4x+56)2 = 672x + 48x^2$

$\Rightarrow 16(x+14)^2 = 16(42x+3x^2)$

$\Rightarrow (x+14)^2 = 42x + 3x^2$

$\Rightarrow x^2 + 28x + 196 = 3x^2 + 42x$

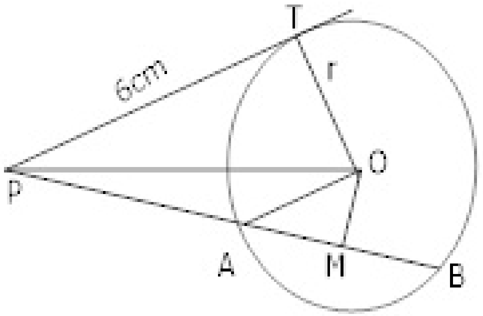
$(x+14)(x-7) = 0$

$x = 7, \quad x = -14$

But $x = -14$ is not possible

$\therefore x = 7$

6. In the given figure, PT is tangent and PAB is a secant. If PT = 6 cm, AB = 5 cm. Find the length PA.



Ans. Join OT, OA, OP. Draw OM $\perp$ AB

Let radius of the circle = r

$\because OT \perp PT$ [$\because$ OT is radius and PT is a tangent]

$\therefore OP^2 = PT^2 + OT^2$ [From right ΔOPT]

$\Rightarrow OP^2 = 6^2 + r^2$

$\Rightarrow OP^2 - r^2 = 36$

$\Rightarrow OP^2 - OA^2 = 36$...........(i) $[OA = OT = r]$

Also from right ΔOMA,

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow OP^2 - 36 = OM^2 + AM^2$$

$$\Rightarrow OP^2 - OM^2 - AM^2 = 36$$

$$\Rightarrow PM^2 - AM^2 = 36$$

$$\Rightarrow (PM + AM)(PM - AM) = 36$$

$$\Rightarrow (PM + AM) PA = 36$$

$$\Rightarrow (PM + MB) PA = 36$$

[$\because AM = MB, \because OM$ bisects AB]

$$(PB)(AP) = 36$$

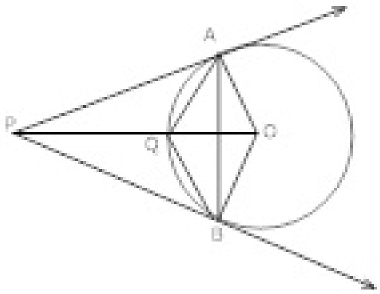
$$\Rightarrow PA(PA + AB) = 36$$

$$\Rightarrow PA^2 + 5PA - 36 = 0$$

$$\Rightarrow (PA + 9)(PA - 4) = 0$$

$\Rightarrow PA = 4.\ or\ PA = -9$ [It cannot be -ve]

7. From a point P two tangents are drawn to a circle with centre O. If OP = diameter of the circle, show that ΔAPB is equilateral.



Ans. Join OP.

Suppose OP meets the circle at Q. Join AQ.

We have

i.e., OP = diameter

$\therefore$ OQ + PQ = diameter

PQ = Diameter – radius [$\because OQ = r$]

$\therefore$ PQ = radius

Thus, OQ = PQ = radius

Thus, OP is the hypotenuse of right triangle

OAP and Q is the mid-point of OP

$\therefore$ OA = AQ = OQ

[$\because$ mid-point of hypotenuse of a right triangle is equidistant from the vertices]

$\Rightarrow \Delta OAQ$ is equilateral

$\Rightarrow \angle AOQ = 60°$

So, $\angle APO = 30° \quad \therefore \angle APB = 2\angle APO = 60°$

Also $PA = PB \Rightarrow \angle PAB = \angle PBA$

But $\angle APB = 60° \therefore \angle PAB = \angle PBA = 60°$

Hence, $\triangle APB$ is equilateral.